Test LOGARITMI / ESPONENZIALI

DOM. 1 Posto $a = 0,21$ $b = 1/5$ $c = \dfrac{1}{\log_2 5}$ si ha:

1) $c < a < b$

2) $a < b < c$

3) $c < b < a$

4) $b < a < c$

5) $a < c < b$

$a = 0,21$

$b = \dfrac{1}{5} = 0,20$ $\Rightarrow b < a$ NO) 1) 2) 5)

$\log_2 5 = x$ tc: $2^x = 5$ $\Rightarrow$ $2 < x < 3$ es. 2,5

$\Rightarrow c = \dfrac{1}{\log_2 5} \cong \dfrac{1}{2,5} \sim \dfrac{2}{5} \sim 0,4$

$c \sim 0,4 > a > b$

RISP. 4)

~~DOM. B1~~

DOM. 3 Sia $a = 2^{\log_2 7 + \log_{\frac{1}{2}} 3}$ allora:

1) $a = 4$

2) $a = 7 + 1/3$

3) $a = 7/3$

4) $a = 21$

5) $a = -21$

CAMBIO base: $\log_{\frac{1}{2}} \rightarrow \log_2 \longrightarrow \dfrac{\log_2 3}{\log_2 1/2} =$

$$- \log_2 3 = \log_2 \left(\frac{1}{3}\right)$$

$$a = \underbrace{2^{\log_2 7}}_{\substack{\shortparallel \\ 7}} \cdot \underbrace{2^{\log_2 (1/3)}}_{\substack{\shortparallel \\ \frac{1}{3}}}$$

$\Rightarrow a = 7/3$

RISP. 3

DOM. 4 Quante delle seguenti uguaglianze sono verificate per ogni n° reale $a>0$, $a \neq \frac{1}{2}$, $a \neq 1$?

• $\log_{2a} a = \frac{1}{2} \Rightarrow (2a)^{1/2} = a \Rightarrow \underbrace{\sqrt{2a} = a}_{\text{solo se } a=2}$

vera

• $\underbrace{(\log_a a^2)}\underbrace{(\log_{a^2} a)} = 1$ VERA sempre

$a^x = a^2$ $(a^2)^y = a^1$ $2 \cdot \frac{1}{2} = 1$

$x = 2$ $2y = 1$

$y = 1/2$

• $\underbrace{\log_{\sqrt{a}} \left(\frac{1}{a}\right)} = -2$

$(\sqrt{a})^x = \frac{1}{a} \Rightarrow a^{1/2\, x} = a^{-1} \Rightarrow \frac{1}{2}x = -1$

$\Rightarrow x = -2$ VERO sempre

• $\underbrace{\log_a \left(\frac{a}{2}\right)} = \frac{1}{2}$

$a^x = \frac{a}{2}$ con $x = \frac{1}{2}$ cioè $\underbrace{\sqrt{a} = \frac{a}{2}}$

solo se $a = 4$

DOM. 13 Il numero $x = \log_2 18$

1) è >4 e <5

2) è $=9$

3) è <0

4) è >5

5) è <4

$$\log_2(9 \cdot 2) = \log_2 9 + \log_2 2 =$$

$$= \underbrace{\log_2 3^2} + \log_2 2 =$$

$$\underbrace{\log_2 8}_{3} < \log_2 9 < \underbrace{\log_2 16}_{4}$$

$$\downarrow$$

è n° compreso tra 3 e 4

$$\Rightarrow \underbrace{\log_2(18) = N(\text{tra } 3 \text{ e } 4) + 1}$$

$$\Downarrow$$

è compreso tra 4 e 5

RISP. 1)

DOM. 6 L'equaz. $\log_{10}(4x) + \log_{10}(9x) = 2$ è verificata per:

EQUAZ. LOGARITMICA

1) $x = 100/13$

2) $x = 20/13$

3) $x = 100/36$

4) $x = 10/6$

5) $x = \pm 10/6$

Porre le C.E:

$$\begin{cases} 4x > 0 \\ 9x > 0 \end{cases} \Rightarrow \boxed{x > 0}$$

Proprietà log:

$$\log_{10}(4x \cdot 9x) = 2$$

$$\log_{10}(36x^2) = 2$$

$$10^2 = 36x^2$$

$$100/36 = x^2$$

$x = \pm 10/6$ però $x = -10/6$ N.A.

$$\Rightarrow S = \{10/6\}$$

RISP. 4)

DOM. 14 Il numero $16^{1/4} + \log_{10}(1000)$ è uguale a

1) 5
2) 0
3) 6
4) 104
5) 7

$$\sqrt[4]{2^4} + \log_{10} 10^3 = 2 + 3 \cdot \log_{10} 10 = 5$$

RISP. 1)

DOM. 19 La disequazione $(1,5)^x < \frac{1}{1,5}$ è verificata per:

1) nessun valore reale di x

2) $x < -1$

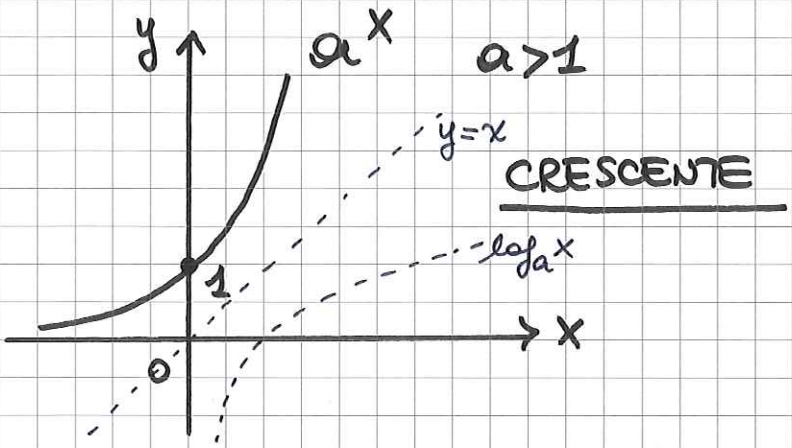
3) $x > -1$

4) $x < 0$

5) $x < 1$

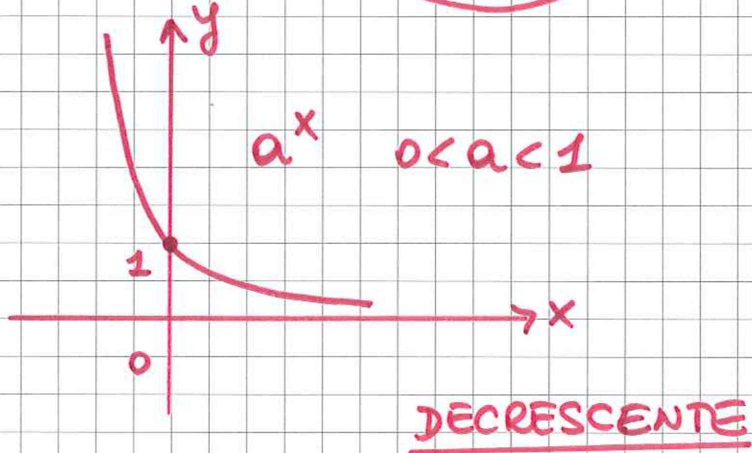
$$(1,5)^x < (1,5)^{-1}$$

Att! base > 1



$\Rightarrow$ $x < -1$

Se fosse stato $\left(\frac{1}{2}\right)^x < \left(\frac{1}{2}\right)^{-1}$ $\Rightarrow$ $x > -1$



RISP. 2)

DOM. 10 — DISEQ. LOGARITHMICA

La disequaz. $2 - |\log_3 x| > 0$ è verificata:

1) $x > 0$
2) $x < \frac{1}{9} \lor x > 9$
3) $x = 1$
4) $\frac{1}{9} < x < 9$
5) $|x| > \log_3 2$

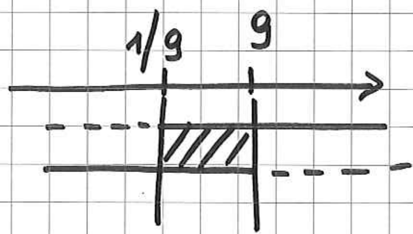
Ricordiamo che: $|x| < 2 \iff -2 < x < 2$

C.E.: $x > 0$

$$|\log_3 x| < 2 \Rightarrow -2 < \log_3 x < 2 \quad \text{ovvero:}$$

$$\begin{cases} \log_3 x > -2 \\ \log_3 x < 2 \end{cases} \quad \begin{cases} \log_3 x > -2 \cdot \log_3 3 \\ \log_3 x < 2 \cdot \log_3 3 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \log_3 x > \log_3 1/9 \\ \log_3 x < \log_3 9 \end{cases} \quad \begin{cases} x > 1/9 \\ x < 9 \end{cases}$$



$$\begin{cases} 1/9 < x < 9 \\ x > 0 \end{cases} \Rightarrow 1/9 < x < 9$$

RISP. 4)

DOM. 20 Trovare il più grande tra i seguenti N°:

$\log_4 15 \qquad \log_{10} 85 \qquad \log_5 23 \qquad \log_2 5 \qquad \log_3 8$

$\underbrace{\log_4 4}_{1} < \log_4 15 < \underbrace{\log_4 16}_{2} \qquad 1 < N° < 2$

$\underbrace{\log_{10} 10}_{1} < \log_{10} 85 < \underbrace{\log_{10} 100}_{2} \qquad 1 < N° < 2$

$\underbrace{\log_5 5}_{1} < \log_5 23 < \underbrace{\log_5 25}_{2} \qquad 1 < N° < 2$

$\log_2 5 > \underbrace{\log_2 4}_{2} \qquad N° > 2 \longleftarrow$

$\underbrace{\log_3 3}_{1} < \log_3 8 < \underbrace{\log_3 9}_{2} \qquad 1 < N° < 2$

RISP. 4)

DOM. 11 Stabilire per quali valori di x, esiste la corrispondente y nella seguente funzione:

$$y = \frac{\sqrt{4-x}}{\log_5 x}$$

1) $0 < x \leq 4$

2) $x \neq 1$

3) $0 < x < 1 \vee 1 < x \leq 4$

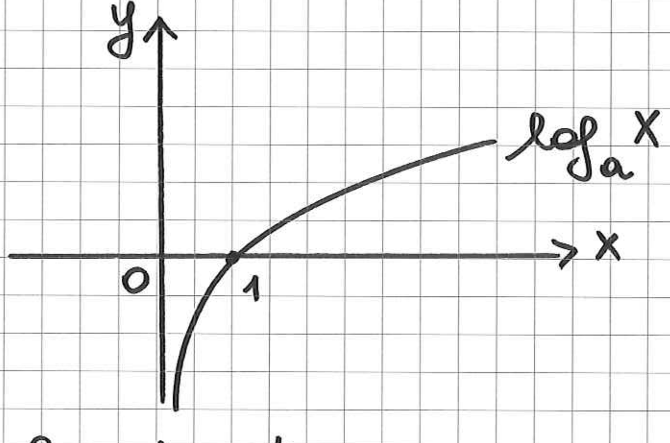
4) $x > 0$

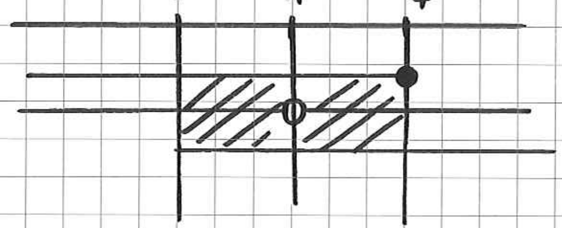
5) $\forall x \in \mathbb{R}$

Trovare il Dominio:

$$\begin{cases} 4 - x \geq 0 & \text{esist. radice pari} \\ \log_5 x \neq 0 & \text{esist. frazione} \\ x > 0 & \text{esist. logaritmo} \end{cases}$$

$$\begin{cases} x \leq 4 \\ x \neq 1 \\ x > 0 \end{cases}$$





$$0 < x < 1 \vee 1 < x \leq 4$$

RISP. 3)